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Go Figure 2001

For Students in grades $7, 8, 9, 10, 11$, and 12

Show your work. You can receive partial credit for partial solutions. Please write all solutions clearly, concisely, and legibly.

The positive integers are the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 \ldots$

1. It is given that $A = 100$, B is A increased by 20%, and C is B decreased by 30%. By how many percent should A be decreased to get C directly from A?

2. When each of the fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ is multiplied by a certain positive integer m, one gets the numbers $30, 20, 15, a, b$. What are m, a and b?

3. In an arithmetic progression, the difference $s - t$ of adjacent terms t, s is fixed. For example, the arithmetic progression $5, 10, 15, 20 \ldots$ has $10 - 5 = 15 - 10 = 20 - 15 = 5$ as the fixed difference.

   (a) Find the 92nd term of the unending arithmetic progression $3, 6, 9, 12 \ldots$ [The three dots after 12 are read as "and so on" to indicate that this unending progression continues with the pattern required of an arithmetic progression.]

   (b) Find the 92nd term of the unending arithmetic progression $2, 5, 8, 11 \ldots$ [Advice: Compare each term with the corresponding term in (a).]

4. How many terms are there in the arithmetic progression $7, 9, 11, 13 \ldots, 467$? This is made up of the consecutive odd positive integers starting with 7 and ending with 467.

5. In this problem, each of A, B and C stands for a different digit chosen from $\{2, 3, 5, 6, 7, 9\}$ and each is fixed throughout the problem. For example, if F stood for 6 then $3F$ would represent 36 and $F4F$ would stand for 646. Find digits A, B, C that make

$$104 \times A8B = 10C440$$

   a correct multiplication statement.

6. There are 24 four-digit positive integers whose digits are $1, 3, 5$, and 7 in some order. For example, 3751 is such an integer. Let S be the set of all 24 of these numbers.

   (a) How many of the 24 numbers in set S have a 5 as the most-significant digit (in the thousands place)?

   (b) What is the smallest number in set S with a 5 in the thousands place?

   (c) How many of the integers in S are smaller than your answer for part b?

7. (a) How many five-digit positive integers have $1, 3, 5, 7, 9$ in some order as their five digits??

   (b) If the numbers to be counted in part (a) were written in increasing order, what would the 27th term be?

8. Consider finite sequences having all of the following properties:

   (i) The first term f is a positive integer with at least two digits.

   (ii) Every term after f is the product of the digits of the term just before it in the sequence.

   (iii) The sequence stops at the first one-digit term.

   For example, $565, 150, 0$ is a three-term sequence of this type since $5\times6\times5 = 150$, $1\times5\times0 = 0$, and 0 has only one digit. Use trial to find the two-digit first term f that leads to a five term sequence satisfying (i), (ii), and (iii).

9. Let a be a positive integer with at least two digits. We define a "cousin" of a to be a positive integer c such that a and c have the same number of digits and differ in exactly one digit. For example, the cousins of 123 are of the forms $d23, 1e3$, and $12f$ where d, e, and f are digits and $d \neq 1, e \neq 2$, and $f \neq 3$.

   (a) 25 has 17 cousins; how many cousins does 364 have?

   (b) List the cousins c of 364 such that $\frac{c}{9}$ is an integer. [Advice: c is an integral multiple of 9 if and only if the sum of the digits of c is an integral multiple of 9.]

10. Let V be the set of positive integers whose eight digits a, b, c, d, e, f, g, h from left to right satisfy the equation $h - g + f - e + d - c + b - a = 0$. Let W be the set of positive integers whose eight digits are $2, 3, 4, 5, 6, 7, 8, 9$ in some order.

    (a) There are $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ integers in W. How many of the integers in W are also in V?

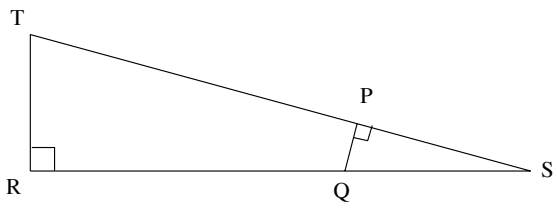
    (b) If the integers that are in both V and W were written in increasing order as a sequence, what would the 4600th integer in this sequence be?

PROBLEM 11 WILL BE USED ONLY TO BREAK TIES.

11. As clearly as you can, justify your answer for Problem 10.

PROBLEMS 12 AND 13 ARE OPTIONAL FOR STUDENTS IN GRADES 7, 8, AND 9.

12. The point P is on the hypotenuse TS of right triangle SRT and Q is on side SR. Both angle SRT and angle SPQ are right angles. The lengths of segments QR, PS and SQ are 47, 24 and 25 units respectively. Find the lengths of segments PT and QP.



13. The net profit of a small company in a given year was \$500,000 minus two items. One of these items was a tax and the other was a bonus for the CEO. To calculate the bonus, subtract the tax from \$500,000 and take 10% of that difference. To calculate the tax, subtract the bonus from \$500,000 and take 5% of that difference. Find the net profit to the nearest dollar.